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THREE DIMENSIONAL SIMULATION OF UT/NERL FREE ELECTRON LASER PROJECT *

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Abstract An FEL project is being carried on at UT/NERL. The transverse effects and the required beam parameters have been estimated by three dimensional FEL simulation. It is found that the electron beam of small radius is essential for our experiment.

INTRODUCTION

A free electron laser (FEL) experiment at the infrared region utilizing an existing rf-linac is planned at UT/NERL (Nuclear Engineering Research Laboratory, University of Tokyo). Table I shows the design parameters for the FEL experiment. The feasibility of the FEL oscillation has been investigated with one dimensional FEL analyses, and the required electron beam quality for the laser oscillation has been clarified. [1]

The laser power saturation during an electron macropulse needs the single pass gain greater than 10%. The one dimensional analyses predict that this value of the gain is attained with the parameters listed in table I.

However the three dimensionality effects caused by transverse distributions and overlapping of the electron and the optical beams may deteriorate the particle-wave interaction and reduce the FEL gain. In the present paper, we present a three dimensional FEL analysis which has been made to estimate the gain reduction due to the transverse effects and clarify the required electron beam quality for the experiment.

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Table I. Design parameters for UT/NERL FEL.

electron beam		wiggler	
energy	γ = 30.0	wavelength	4.4 cm
peak current	7 A	wiggler parameter	K = 1.02
beam radius	2 mm	number of period	25
macropulse	4.5μsec		
energy spread	< 1%	laser wavelength	50.5 μm

FEL ANALYSIS

FEL equations in phase space

One dimensional FEL equations are summerized by the following equations:

$$\frac{\partial E_\perp}{\partial t} = \frac{\mu_0 e c^3 K n_e}{\sqrt{2}} \left\langle \frac{\sin\Psi_j}{\gamma_j} \right\rangle_j \tag{1}$$

$$E_\perp \frac{\partial \phi}{\partial t} = \frac{\mu_0 e c^3 K n_e}{\sqrt{2}} \left\langle \frac{\cos\Psi_j}{\gamma_j} \right\rangle_j \tag{2}$$

$$\frac{d\gamma_j}{dt} = -\frac{eKE_\perp}{\sqrt{2}mc} \frac{\sin\Psi_j}{\gamma_j} \tag{3}$$

$$\frac{d\Psi_j}{dt} = c\left[k_w - k\,\frac{1+K^2}{2\gamma_j^2}\right] + \frac{d\phi}{dt}\ , \tag{4}$$

where $E_\perp$ and ϕ are the amplitude and the phase of the optical field $\mathbf{E} = E_\perp e^{i(kz-\omega t+\phi)}$, K is the wiggler parameter, k_w is the wiggler wave number, k is the optical wave number, γ is the dimensionless electron energy, $\Psi = (k+k_w)z-\omega t+\phi$ is the electron phase and $\langle\ \rangle_j$ means averaging over ensemble electrons.

These equations show that the time evolution of the optical field can be calculated by tracking the electrons in the γ–Ψ phase space. This formulation is suited for a numerical simulation, since it includes only the slow varying parts of the field, but it cannot be extended straightforwardly to the three dimensional form. Therefore we have derived the relation between the electron motion and the optical field in more general form that is suitable to the three dimensional calculations.

FEL equations in real space

The energy loss of the electron beam, $\Delta\Phi$, and the energy gain of the optical beam, ΔP, are obtained from equation (1) and (3), respectively:

$$\Delta\Phi = \frac{I}{e\Sigma}\Delta\gamma mc^2 = \frac{ec^2Kn_eE_\perp}{\sqrt{2}}\left\langle\frac{\sin\Psi_j}{\gamma_j}\right\rangle_j \Delta t \tag{5}$$

$$\Delta P = \sqrt{\varepsilon_0/\mu_0}\, E_\perp\Delta E = \frac{ec^2Kn_eE_\perp}{\sqrt{2}}\left\langle\frac{\sin\Psi_j}{\gamma_j}\right\rangle_j \Delta t \;, \tag{6}$$

where I is the electron beam current and Σ is the cross-sectional area of the electron beam. It is shown that the energy loss of the electron beam is equal to the energy gain of the optical beam. This means the energy conservation law in the system and also is the results of the electromagnetic reciprocity theorem — the energy of the stimulated radiation by an electron onto a given optical mode is equal to the work done by the mode on the electron. Equation (1) and (3), then, give the equation which directly relates the energy gain of the optical field to the energy loss of the electrons:

$$\frac{\partial E_\perp}{\partial t} = -\frac{\mu_0 mc^4 n_e}{E_\perp}\left\langle\frac{d\gamma_j}{dt}\right\rangle_j \tag{7}$$

$$\frac{d}{dt}(mc^2\gamma_j) = e\mathbf{v}\cdot\mathbf{E} \;, \tag{8}$$

where $\mathbf{E}$ should be consistent with the Maxwell equations under the given boundary conditions.

The time evolution of the optical phase can also be obtained from equation (2) and (3) as follows;

$$\frac{\partial \phi}{\partial t} = -\frac{\mu_0 mc^4 n_e}{E_\perp^2}\left\langle\frac{d\tilde{\gamma}_j}{dt}\right\rangle_j \;, \tag{9}$$

where $d\tilde{\gamma}/dt$ is the imaginary energy variation of the electrons due to the imaginary wave which has the same amplitude as the optical wave and the phase advanced $\pi/2$, $\tilde{\mathbf{E}} = \mathbf{E}e^{i\pi/2}$, and

$$\frac{d}{dt}(mc^2\tilde{\gamma}_j) = e\mathbf{v}\cdot\tilde{\mathbf{E}} \;, \tag{10}$$

where $\tilde{\gamma}_j$ is taken to be equal to γ_j .

Thus, the time evolution of the amplitude and the phase of the optical field is directly related with the energy variation of the electrons. This formulation describes the FEL mechanism in a physical sense. Moreover, it is independent of a system of coordinates. So it is easy to be applied to three dimensional calculations.

Three dimensional FEL gain calculations

We have developed a three dimensional FEL simulation code based on the formulation derived in the previous section. This code can take account of such realistic situations which cannot be handled by one dimensional analysis, as inhomogeneous spatial distribution of the optical beam, overlapping between the electron beam and the optical beam, and the variation of the electron density due to the betatron oscillation.

For the gain calculations using equations (7) and (8), the wiggler and the optical field that determine the electron motion should be given throughout the interaction region. The magnetic field of a Halbach type wiggler is obtained as the summation of the fields all the pole surfaces generate. The field generated by each magnet piece is calculated by integrating the magnetic charge distributed uniformly over the pole surface. This integration of the magnetic charge have been performed analytically to reduce the computation time.

The spatial distribution of the optical field depends on the configuration of FEL; while in an FEL oscillator the field can be expanded into resonator modes, the field distribution in a high-gain FEL amplifier is determined by the shape of the injected light beam and the interaction between the light and the electron beam. We have used the FEL oscillator configuration which is generally used for an rf-linac based FEL. The optical field, the solution of the Maxwell equations with the resonator boundary conditions, is represented by the resonator modes which consist of the fundamental Gaussian mode and the higher order modes.

RESULTS AND DISCUSSIONS

First, the results of the three dimensional calculations have been compared with that of the one dimensional theoretical analyses. In these calculations, we used the "real space formulation", but ignored the transverse effects and assumed the plane wave optical field for the comparison. Figure 1 shows the calculation results using the values of the parameters given in table I (the beam energy spread is not included). Both results are in good agreement with each other.

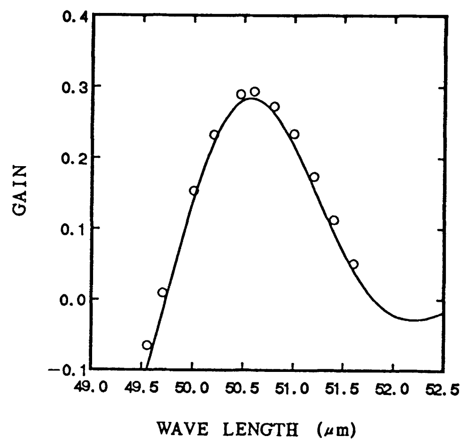

FIGURE 1 Calculated FEL gain. Solid line: 1-D theory, circle: 3-D simulation.

Calculations including the spatial distribution of the electron and the optical beam have been made to estimate the gain reduction by the transverse effects. The electron beam of the Gaussian shape ($2\sigma=2mm$, and $1mm$) and the optical beam of the fundamental resonator mode have been assumed. The calculated gain of the case with the 2mm radius beam is much less than that of the one dimensional analysis, as shown in table II. This gain reduction is due to the transverse effects. The wiggler field varies transversely, i.e. as a function of the distance from the center axis of the wiggler, so the resonant condition no more holds for the outer off-axis electrons. The electrons energy at the exit of the wiggler as a function of the distance from the center axis are presented in figure 2, which clearly shows the outer electrons do not contribute to the optical beam amplification. The calculations show that the electron beam whose radius is less than 1mm is required for our experiment to avoid detrimental three dimensional effects.

TABLE II. Calculated FEL gain.

	electron beam radius	gain/pass
1-D	2mm	28%
3-D	2mm	4%
3-D	1mm	29%

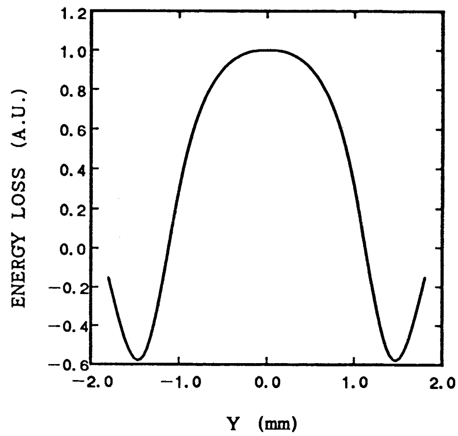

FIGURE 2 Energy loss of the electrons calculated by the 3-D simulation.

CONCLUSIONS

An FEL formulation in the real space has been derived. Three dimensional calculations based on this formulation have been performed and it has been shown that the electron beam radius should be less than 1mm to guarantee the required FEL gain for our experiment.

REFERENCES

1. H. Ohashi, R. Hajima, M. Kishimoto, H. Kobayashi and S. Kondo, Proc. of the 12th Linear Acc. Conf. in Japan,pp.123-125,(1987).